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On the Exchange Part of the Single-Particle Spin-Orbit Potential Following from the Nucleon-Nucleon Spin-Orbit Interaction

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The exchange part of the single-particle spin-orbit potential following from a two-body spin-orbit potential is calculated. Scheerbaum's approximation of setting the exchange part equal to the direct part is investigated.

The single-particle (s.p.) spin-orbit (s.o.) potential resulting from the short-range two-particle (t.p.) s.o. potential is not expected to be essentially influenced by curvature effects [1]. Thus its form can be derived by using the simple model of the one-dimensional nucleus with its plane surface.

The direct part V_D of the s.p.s.o. potential can easily be calculated using the standard methods:

$$V_D = \sum_{j=1}^{\infty} M_j \, n^{(2j-1)}(z) \, (\sigma_x k_y - \sigma_y k_x) . \tag{1}$$

M_j are the moments of the nucleon-nucleon s.o. potential, n is the nuclear density, $\boldsymbol{k}$ the wave vector. The operator $(\sigma_x k_y - \sigma_y k_x)$ is the equivalent of the operator $-(\boldsymbol{l}\cdot\boldsymbol{\sigma})/r$ in the case of a spherical nucleus.

Scheerbaum [2] has given arguments favoring the approximation to set the exchange part V_E equal to V_D. In the present paper the exchange contribution V_E is calculated for the system of semi-infinite nuclear matter. Scheerbaum's first approximation — used also elsewhere, e. g. in [3] — is verified; the higher order corrections are given in a closed form.

The s.p. wave function in the semi-infinite medium is denoted by its quantum numbers $\boldsymbol{k}$ (the wave number), $\varkappa = \pm 1$ (the spin orientation), and t_3 (the isospin). Introducing the standard normalizing volume Ω (see e. g. Ref. [4]) the spatial part of the s.p. wave function in the semi-infinite nuclear system is given by

$$\psi_{\boldsymbol{k}}(\boldsymbol{r}) = \left(\frac{4}{\Omega}\right)^{1/2} \varphi_{k_z}(z) \, e^{i(k_x x + k_y y)} . \tag{2}$$

The appropriate spin functions are given in Ref. [5].

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The diagonal matrix elemet of the s.p.s.o. potential is defined by

$$\langle \boldsymbol{k}\varkappa t_3 | V | \boldsymbol{k}\varkappa t_3 \rangle = \langle \boldsymbol{k}\varkappa t_3 | V_D + V_E | \boldsymbol{k}\varkappa t_3 \rangle \tag{3}$$

with

$$\langle \boldsymbol{k}\varkappa t_3 | V_D | \boldsymbol{k}\varkappa t_3 \rangle = \sum_{\boldsymbol{k}',\varkappa',t_3'} \langle \boldsymbol{k}\varkappa t_3 ; \boldsymbol{k}'\varkappa' t_3' | V_{12} | \boldsymbol{k}\varkappa t_3 ; \boldsymbol{k}'\varkappa' t_3' \rangle , \tag{3 a}$$

$$\langle \boldsymbol{k}\varkappa t_3 | V_E | \boldsymbol{k}\varkappa t_3 \rangle = -\sum_{\boldsymbol{k}',\varkappa',t_3'} \langle \boldsymbol{k}\varkappa t_3 ; \boldsymbol{k}'\varkappa' t_3' | V_{12} | \boldsymbol{k}'\varkappa' t_3' ; \boldsymbol{k}\varkappa t_3 \rangle , \tag{3 b}$$

$$V_{12} = (V_{12}^{3+} P^{3+} + V_{12}^{3-} P^{3-}) \, \boldsymbol{l}\cdot\boldsymbol{S} .$$

V^{3+} and V^{3-} are the spatial parts of the s.o. potential for the corresponding t.p. states; the operators P^{3+} and P^{3-} are the respective projection operators. Carrying out the isospin summation in the exchange part (3 b) corresponds in the usual way to the replacement

$$\sum_{t_3'} \langle t_3 t_3' | V_{12}^{3+} P^{3+} + V_{12}^{3-} P^{3-} | t_3' t_3 \rangle = \langle t_3 | 4 J^E | t_3 \rangle ,$$

$$4 J^E = \tfrac{3}{2} V_{12}^{3-} - \tfrac{1}{2} V_{12}^{3+} . \tag{4}$$

After the spin summation in Eq. (3 b), the exchange part V_E has the following form

$$\langle \boldsymbol{k}\varkappa t_3 | V_E | \boldsymbol{k}\varkappa t_3 \rangle = -2 \langle \varkappa | \boldsymbol{\sigma} | \varkappa \rangle \cdot \frac{\Omega}{(2\pi)^3} \int_{\vartheta} \mathrm{d}^3\boldsymbol{k}' \iint \mathrm{d}^3\boldsymbol{r}_1 \, \mathrm{d}^3\boldsymbol{r}_2 \, \psi_{\boldsymbol{k}}^*(\boldsymbol{r}_1) \, \psi_{\boldsymbol{k}'}^*(\boldsymbol{r}_2) \, J^E(r_{12}) \, (\boldsymbol{r}_1 - \boldsymbol{r}_2) \times \frac{1}{i} (\nabla_1 - \nabla_2) \, \psi_{\boldsymbol{k}'}(\boldsymbol{r}_1) \, \psi_{\boldsymbol{k}}(\boldsymbol{r}_2) , \tag{5}$$

ϑ being the Fermi hemisphere domain of integration. The z-component of the spatial part of expression (5) is of no relevance since the z-component of the spin part vanishes. With the mixed density [6],

$$n(\boldsymbol{r}_1, \boldsymbol{r}_2) = n(z_1, z_2, \sqrt{x^2 + y^2})$$

$$= \frac{1}{2\pi^2} \int_0^{k_F} \mathrm{d}k_z \, \varphi_{k_z}(z_1) \, \varphi_{k_z}(z_2) \, (k_F^2 - k_z^2) \, \frac{2 J_1(t)}{t} , \tag{6}$$

$$t = \sqrt{x^2 + y^2} \sqrt{k_F^2 - k_z^2} , \quad x = x_1 - x_2 , \quad r = |\boldsymbol{r}_1 - \boldsymbol{r}_2| ,$$

the spatial part of Eq. (5) is evaluated explicitly

$$\langle \boldsymbol{k}\varkappa t_3 | V_E | \boldsymbol{k}\varkappa t_3 \rangle = (16/\Omega) \, \langle \varkappa | \boldsymbol{\sigma} | \varkappa \rangle \cdot \iint \mathrm{d}^3\boldsymbol{r}_1 \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z_2 \, J^E(r) \, \varphi_{k_z}(z_1) \begin{pmatrix} y \cdot \sin k_y y \cdot \cos k_x x \\ -x \cdot \sin k_x x \cdot \cos k_y y \end{pmatrix} \varphi_{k_z}(z_2) \frac{\partial}{\partial z_1} n(\boldsymbol{r}_1, \boldsymbol{r}_2) . \tag{7}$$

In order to derive a manageable expression from Eq. (7) for the exchange part of the s.p.s.o. potential the term

$$\varphi_{k_z}(z_2)\,\frac{\partial}{\partial z_1}\,n(\boldsymbol{r}_1,\boldsymbol{r}_2)$$

is expanded up to the second order by using the Taylor series of the wave function $\varphi_{k_z}(z_2)$ about the point z_1 and the series expansion of the mixed density according to formula (6). Specialising $J^{\mathrm{E}}(r_{12})$ to the Gaussian form $J^{\mathrm{E}}(r)=A\exp\{-\alpha^2 r^2\}$ we obtain for the diagonal part of (7) after some elementary integrations the following expression:

$$V_{\mathrm{E}}=\frac{\pi^{3/2}A}{\alpha^5}\exp\{-(k_x^2+k_y^2)/(4\alpha^2)\}\left[n'(z)+h_1(z)+\frac{k_x^2+k_y^2}{4\alpha^2}\,h_2(z)+\frac{1}{4\alpha^2}\frac{\partial}{\partial z}\,n'(z)\,\frac{\partial}{\partial z}\right](\sigma_x k_y-\sigma_y k_x). \tag{8}$$

The functions $h_j(z)$ vanish in the inner part of semi-infinite nuclear matter. They are defined by

$$h_1(z)=\frac{1}{\pi^2\alpha^2}\left[-\int_0^{k_{\mathrm{F}}}\varphi_{k_z}(z)\;\varphi'k_z(z)\;(k_{\mathrm{F}}^2-k_z^2)^2\,\mathrm{d}k_z-\tfrac{1}{4}\int_0^{k_{\mathrm{F}}}\varphi'k_z(z)\;\varphi''k_z(z)\;(k_{\mathrm{F}}^2-k_z^2)\,\mathrm{d}k_z\right]$$

$$h_2(z)=\frac{1}{2\pi^2\alpha^2}\int_0^{k_{\mathrm{F}}}\varphi_{k_z}(z)\;\varphi'_{k_z}(z)\;(k_{\mathrm{F}}^2-k_z^2)^2\,\mathrm{d}k_z\,.$$

The second order terms contained in Eq. (8) are of the order $k_{\mathrm{F}}^2/(4\alpha^2)$ relative to the first order approximation given by the leading term of Eq. (8), which is simply proportional to $n'(z)$.

Formula (8) can be conveyed to spherical nuclei for a calculation of the exchange contribution to the s.o. splittings. The functions $g_j(z)$ may be calculated for the semi-infinite system by using e. g. the conventional linear s.p. potential to generate the wave functions $\varphi_{k_z}(z)$. Then we may proceede to finite spherical nuclei by making the appropriate replacements

$$z\to r\,,\qquad (k_x^2+k_y^2)/(4\alpha^2)\to \boldsymbol{l}^2/(4\alpha^2 r^2),\qquad \sigma_x k_y-\sigma_y k_x\to -\boldsymbol{l}\cdot\boldsymbol{\sigma}/r\,.$$

Thus, for spherical nuclei the exchange part V^{E} of the s.p.s.o. potential is given by

$$V_{\mathrm{E}}=-\frac{\pi^{3/2}A}{\alpha^5}\exp\{-\boldsymbol{l}^2/(4\alpha^2 r^2)\}\,\frac{1}{r}\left[n'(r)+g_1(r)+\frac{\boldsymbol{l}^2}{4\alpha^2 r^2}\,g_2(r)+\frac{1}{4\alpha^2}\frac{\partial}{\partial r}\,n'(r)\,\frac{\partial}{\partial r}\right]\boldsymbol{l}\cdot\boldsymbol{\sigma}. \tag{9}$$

For the calculation of the s.o. splittings the diagonal matrixelement of V_{E} between oscillator wave functions has to be taken. The operator $\boldsymbol{l}^2$ can then be replaced by the number $l(l+1)$.

If the 3^+ part of the t.p.s.o. potential may be neglected the exchange part V_{E} equals in first inhomogeneity order the direct part V_{D}.

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